A physically-based failure analysis framework for fiber-reinforced composite laminates under multiaxial loading

Jian Deng, Zhenjun Hong, Qiaozhi Yin, Tian Jian Lu

1. Introduction

Fiber reinforced composites are widely accepted as efficient alternatives for designing light-weight and high-performance structures, yet theoretical prediction of failure for such composites is still a challenging task with uncertainties and controversies. In this work, a new physically-based failure analysis framework is proposed to predict both intralaminar failure onset and strengths for composite laminates under general stress states, with interactive and coupling effects of stresses fully considered. The in situ strengths are introduced using the simplified fracture mechanics-based approximation formula where the constraining effects of both the adjacent plies and embedded laminar thickness are considered. The proposed framework is validated by comparing predictions with existing experimental data. Both initial and final failure envelopes are well predicted for unidirectional and multi-directional laminates under multiaxial loads. Stress-strain responses are also well captured, further illustrating the influence of in situ strengths on failure initiation.

Keywords:
Composite laminates
Failure criteria
Strengths
Fracture plane

ABSTRACT

Fiber reinforced composites are widely accepted as efficient alternatives for designing light-weight and high-performance structures. However, the cost reduction by advanced manufacturing techniques, carbon and glass fiber reinforced plastics (CFRP, GFRP) are highly demanded in a wide range of industries ranging from aerospace, naval vehicles, auto-mobile and civil construction to mention a few. Nevertheless, composite laminates are generally over-designed to ensure reliable structural performance, as theoretical predictive models of damage and failure mechanisms in composites have not been fully established yet and are still under development. Uncertainties and controversies remain despite of large efforts devoted to this subject [1–13].

During the continuous development of failure criteria for composite laminates in the last thirty years, two main divisions are classified, namely, non-physically based and physically based criteria. Initiated by von Mises stress theory of isotropic materials, a composite laminate is considered as a single orthotropic material in non-physically based criteria. They are usually formulated in uniform polynomials [3,14], so that local failure mechanisms are not considered under combined stress states, thus enhancing the efficiency in damage tolerance assessment during the preliminary design stage of composite structures. On the other hand, the physically based criteria are generally formulated with separated expressions, accounting for different failure mechanisms. For this reason, more detailed information of local damage can be provided. Specifically, intralaminar failure of composite laminates can be mainly divided into longitudinal failure (e.g., fiber rupture and kinking) and transverse or inter-fiber failure which is mostly initiated by matrix cracking.

Hashin [1,15] initiated the work on physical based failure criteria and introduced interactive criteria to directly determine the failure modes for unidirectional (UD) laminates. Most successive failure theories more or less referred to Hashin’s work, but the stress interactions do not always correlate well with experimental data, especially for transverse matrix failure [3,16]. On the basis of Mohr-Coulomb failure theory and existing knowledge on damage mechanisms, Puck and co-authors [2,17] proposed the concept of fracture plane and fracture
angle to particularly deal with transverse compression failure. During the three times of World-Wide Failure Exercises (WWFE), the fracture plane concept was highly recommended by many researchers for predicting inter-fiber failure of laminates under combined stresses \([3–5,18]\). Davila et al. \([7]\) further developed a novel analysis framework for predicting the intralaminar failure of UD laminates based on Hashin’s interactive stress theory and Puck’s fracture plane concept.

Given that the initial geometric configurations of laminates are thin and slender, the plane stress hypothesis has been extensively employed. However, three dimensional stress states are more practically applied where the out-of-plane stresses play an important role in failure initiation of laminated structures \([6,19]\). This is due to the fact that general stress states are quite common in multi-directional engineering structures having geometric discontinuities, e.g., free edges, open holes and inserts, asymmetric stiffened and sandwich panels. On the other hand, the basic linear form of Puck’s theory was preferred to establish failure analysis strategies. Nevertheless, further investigations showed that the predicted strengths using the linear form were generally greater than experimental data, especially for the case under high compressive load \([4]\). To handle these general stress states and limitations, improvements and extensions in failure modeling frameworks were further advanced, with additional consideration on material properties and geometric information using the concept of fracture plane \([12,20]\).

It should be mentioned that most recent failure models adopted the maximum stress or strain criterion to address fiber tensile failure due to its simplicity and applicability. However, its prediction accuracy was still controversial under certain combined stress states \([5,21]\). Recently, researchers from Imperial College London and KU Leuven initiated a project, called the Fiber Break Models for Designing novel composite microstructures and applications (FibreMoD), to further study the tensile response of UD laminates and benchmark the positive and negative points of several tensile failure criteria \([22–25]\). In terms of the \textit{in situ} effects in failure predictions, the laminar transverse tensile and in-plane shear strengths were simply multiplied by specific coefficients in WWFE-II \([18]\). Nevertheless, this approximation was too aggressive for the outermost laminar where no constraint existed at the free side. This could lead to invalid failure predictions of multi-directional laminates. Herein, a general theoretical formulation for \textit{in situ} strengths is demanded, with full consideration of the constraining effect of adjacent plies and the embedded laminar thickness.

From the literature review, it is seen that the progression on theoretical predictive models of damage and failure mechanisms for composites is clear from past researches, however, this has been scattered over numerous papers. Therefore, in the current study, the important development on the theoretical predictive models is firstly summarized and discussed. As reliable theoretical predictive models for composites under general loadings are still in the development stage, the other objective of the present investigation is to develop a physically-based failure analysis framework to predict the intralaminar failure onset and strengths for composite laminates under general stress states. Under multiaxial loading, interactive and coupling effects of stresses are carefully considered. The \textit{in situ} strengths are introduced into the failure model using the simplified fracture mechanics-based approximation formula, with the constraining effects of both adjacent plies and embedded laminar thickness considered. A fiber kinking formulation is established with initial manufacturing defects considered for laminates used in practical engineering. To predict the strengths, a simplified degradation scheme of material properties is further proposed, differing various failure modes. Validations are then conducted on both uni-directional and multi-directional laminates subjected to uniaxial and multiaxial loads. Finally, relevant discussions and conclusions are drawn.

2. Longitudinal tension failure modeling

Under uniaxial tension, the simple maximum stress failure criterion is commonly used to predict the tensile strength of UD laminates. However, under complex loading, it is still controversial whether this criterion can accurately predict fiber rupture, since neither superposition nor coupling of stresses are included in the criterion \([5,21,26]\). Hashin \([1,15]\) suggested that the shearing behavior in fiber tension failure should be carefully considered. With fiber tension failure assumed as the interaction of normal and shear stresses on the fracture surface, the quadratic superimposition formulation is proposed, as follows:

\[
f_f = \left(\frac{\sigma_{11}}{X_T}\right)^2 + \left(\frac{\tau_{12}}{S_{12}^{f}}\right)^2 + \left(\frac{\tau_{13}}{S_{13}^{f}}\right)^2 = 1, \quad (\sigma_{11} \geq 0)
\]  

where \(X_T\) is the tensile strength in the fiber direction, \(S_{12}^{f}\) and \(S_{13}^{f}\) represent the axial shear strengths against fracture across the fibers due to pure shear stress, and \(f_f\) is the tensile failure factor.

Comparison of failure of the quadratic Hashin criterion and the maximum stress criterion is shown in Fig. 1. The experimental data were obtained from CFRP laminates (T300/BSL914C) under combined longitudinal tension and in-plane shearing from the WWFE-I \([27]\). The material properties are illustrated in Table 1. It can be seen that the quadratic criterion correlates well with data points for \(\sigma_{11}/X_T > 0.5\), namely, the combined shearing and tension condition supporting an interaction of normal and shear stress. On the other hand, the simple maximum stress criterion can only give a good prediction for greater values of \(\sigma_{11}/X_T\) (near 1.0), namely, near pure tension. This indicates that the interaction of stresses needs to be carefully considered, and the quadratic criterion gives better predictions for multiaxial stress states. It is worth noting that data enclosed by an ellipse, shown in Fig. 1, represent the matrix shear failure and thus are excluded for the verification of the fiber rupture failure criterion.

### Table 1

**Mechanical properties of T300/BSL914C unidirectional laminates \([27]\).**

<table>
<thead>
<tr>
<th>(E_{11})</th>
<th>(E_{22})</th>
<th>(G_{12})</th>
<th>(G_{13})</th>
<th>(G_{23})</th>
<th>(\nu_{12})</th>
<th>(\nu_{13})</th>
<th>(\nu_{23})</th>
<th>(\phi_f^{a})</th>
</tr>
</thead>
<tbody>
<tr>
<td>138 GPa</td>
<td>11 GPa</td>
<td>5.5 GPa</td>
<td>5.58 GPa</td>
<td>0.28</td>
<td>0.06</td>
<td>53</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(X_1)</td>
<td>(X_C)</td>
<td>(Y_1)</td>
<td>(Y_C)</td>
<td>(S_{12})</td>
<td>(S_{13})</td>
<td>(S_{23})</td>
<td>(S_{12}^{f} = S_{13}^{f})</td>
<td></td>
</tr>
<tr>
<td>1500 MPa</td>
<td>900 MPa</td>
<td>27 MPa</td>
<td>200 MPa</td>
<td>80 MPa</td>
<td>70 MPa</td>
<td>130 MPa</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3. Inter-fiber failure modeling

On the basis of Mohr-Coulomb fracture theory and existing knowledge on damage mechanisms, Puck and co-authors [2,17] proposed the concept of fracture plane. In the case of brittle matrix fracture or inter-fiber failure (IFF) of laminates, a fracture plane parallel to the fiber direction is formed. It is assumed that normal and shear tractions acting on a fracture plane cause the failure. Fig. 2 illustrates the fracture plane and local coordinates as well as tractions acting on the plane. The angle between the fracture plane and thickness direction is defined as the fracture angle, $\beta_p$. Regarding the stress states and fracture angle, the IFF is categorized into three different fracture modes as shown in Fig. 3. For normal tension or low level of compression interacting with shearing, the fracture plane is perpendicular to the loading direction with $\beta_p = 0^\circ$, namely, Mode A and B. This further supports Hashin’s inter-action formulations. For a higher level of transverse compression (Mode C), the fracture plane is inclined increasingly with the value of $|\sigma_{22}/\sigma_{11}|$. Furthermore, it was noted in tests that fracture occurred in the inclined plane $\beta_p = 57^\circ$, instead of the plane where maximum shear stress was located ($\pm 45^\circ$), for CFRP unidirectional laminates under transverse compression [6,17]. This can be explained by the fact that the compressive stress and friction on the potential fracture plane cause the shift of the inclined angle. This phenomenon further consolidates the concept of fracture plane in Puck’s failure criteria. The basic form for inter-fiber failure initiation in light of Puck’s theory is given as follows:

IFFT: \[ \left( \frac{\sigma_n}{\sigma_T} \right)^2 + \left( \frac{\tau_{nt}}{\tau_{nt}} \right)^2 = 1, \quad \alpha_n > 0 \]

IFFC: \[ \left( \frac{\tau_{nl}}{\tau_{nl}} \right)^2 + \left( \frac{\tau_{nt}}{\tau_{nt}} \right)^2 = 1, \quad \alpha_n < 0 \]  

where $\sigma_n$, $\tau_{nt}$ and $\tau_{nl}$ are the tractions on the potential fracture plane as shown in Fig. 2, $\beta_{nl}$ and $\beta_{nt}$ are the inclination parameters of contour lines of the fracture body [28] that are used for representing the friction effect on the fracture plane. $\sigma_{nl}$ and $\tau_{nl}$ are the in-plane and transverse fracture resistance of the fracture plane, while $R^+_n$ stands for the tension fracture resistance. IFFT and IFFC represent the inter-fiber tension and compression, respectively.

These tractions on the fracture plane appearing in Eq. (2) can be derived from stresses in material coordinates 1-2-3 using Eq. (3), as:

\[
\begin{align*}
\sigma_n &= \sigma_{22}\cos^2\beta_p + \sigma_{11}\sin^2\beta_p + 2\tau_{23}\cos\beta_p\sin\beta_p \\
\tau_{nl} &= \sigma_{11}\cos\beta_p + \tau_{23}\sin\beta_p \\
\tau_{nt} &= -\sigma_{22}\cos\beta_p\sin\beta_p + \sigma_{11}\cos\beta_p\sin\beta_p + \tau_{23}(\cos^2\beta_p - \sin^2\beta_p)
\end{align*}
\]  

Fracture resistance can be determined using material strength obtained at the corresponding single stress state. For uni-axial transverse tension ($\sigma_{22} > 0$) or pure in-plane shear ($\tau_{23} = 0$), the formed fracture plane is parallel to the principle material direction, so that the fracture angle is settled as $\beta_p = 0^\circ$ (only $\sigma_{22}$ is applied) or $\beta_p = 90^\circ$ (only $\tau_{23}$ is applied). Therefore, $R^+_n$ and $R_{nl}$ can be determined by transverse tension and in-plane shear strengths ($Y_T$ and $S_{13}$), namely,

\[
\begin{align*}
R^+_n &= Y_T \\
R_{nl} &= S_{13}
\end{align*}
\]  

However, $R_{nl}$ cannot be directly obtained. For a UD CFRP under the single stress of $\sigma_{13}$, the fracture angle is approximately $45^\circ$ [29], not the same as the one where $\tau_{23}$ acts alone. Therefore, one cannot simply use the transverse shear strength $S_{13}$ to represent the transverse fracture resistance $R_{nl}$. The inclined fracture plane indicates that the failure is caused by normal tension, which can additionally be derived from Eq. (3) with $\sigma_n > 0$. Given that shearing behavior contributes much more to the failure of UD laminates under transverse compression [2,29], uni-axial transverse compression can be used to determine $R_{nl}$.

For a single stress state $\sigma_{22} < 0$, the tractions on the fracture plane can be obtained using Eq. (3) where $\tau_{nt} = 0$. Then the IFF initiation criterion can be simplified as:

\[ \tau_{nl} + p_n\sigma_n = R_{nl}, \quad \sigma_n < 0 \]  

Furthermore, the stress states of transverse compression (direction-2) failure on the fracture plane can be illustrated in the form of Mohr circle, with Eq. (5) represented by the failure envelope $l_\beta$ as shown in Fig. 4. The Mohr circle of pure transverse compression is tangent to the
failure envelope $l_{ef}$ at point $A$ where the stress state satisfies the failure initiation criterion. This reveals the relationship between the inclination parameter $p_{nt}$ and the fracture angle $\beta_p$, which is

$$p_{nt} = -\tan q_0 = -\cot 2\beta_p$$

(6)

Along with stress states at point $A$, submitting Eqs. (3) and (6) into Eq. (5), one can derive $R_{nt}$ as follows:

$$Y_C \cos^2 \beta_p \sin^2 \beta_p + Y_C \cot^2 \beta_p \rho_0 = R_{nt} \Rightarrow R_{nt} = 2Y_C \cot^2 \beta_p$$

(7)

And $p_{nt}$ is simply estimated using Eq. (8), as

$$p_{nt} = R_{nt} / R_{nt}$$

(8)

Further investigations show that the predicted strengths using the basic form (Eq. (5)) is generally greater than experimental data especially when high compression traction ($\sigma_c < 0$) acts on the potential fracture plane. Corrections are made by Puck and co-authors [17], namely,

$$\text{IFFT:} \quad \left[ \left( \frac{1}{\kappa^2} - \frac{I_1}{I_2} \frac{\sigma_c}{\kappa} \right)^2 + \left( \frac{\sigma_c}{\kappa} \right)^2 + \frac{I_2}{\kappa} \sigma_c \right] = 1, \quad \sigma_c \geq 0$$

$$\text{IFFC:} \quad \left[ \frac{I_2}{\kappa} \sigma_c \right]^2 + \left( \frac{\sigma_c}{\kappa} \right)^2 + \frac{I_2}{\kappa} \sigma_c = 1, \quad \sigma_c < 0$$

(9)

The main difference between the two formulations lies in the description of the Mohr-Coulomb fracture behavior. In Eq. (5), the effective shear fracture resistance ($R_{nt} - p_{nt} \sigma_c, R_{nt} - p_{nt} \sigma_c$) increases linearly with the normal traction $\sigma_n$ while parabolic relation is represented in Eq. (9). Fig. 5 illustrates the Mohr circle to determine the fracture parameters in the parabolic form. Therefore, the fracture resistance parameters can be derived in an analogous way as the case of the linear failure criteria presented above, as:

$$R_{\psi n}^A = \frac{Y_C}{2(1 + P_{11})}$$

(10)

$$R_{\psi n}^{fc} = Y_C$$

(11)

$$R_{\psi n}^{fr} = S_{12}$$

(12)

The inclination parameters are determined using the following equations:

$$\frac{P_{22}^{nt}}{R_{nt}^{A}} = \cos^2 \psi + \sin^2 \psi \cos \tau_{nt}, \quad i = t, c$$

(14)

$$\cos^2 \psi = \frac{\tau_{22}}{\tau_{22}^{2} + \tau_{11}^{2}}, \quad \sin^2 \psi = \frac{\tau_{11}}{\tau_{22}^{2} + \tau_{11}^{2}}$$

(15)

For typical brittle CFRP/epoxy and GFRP/epoxy UD laminates, the values of inclination parameters are recommended as listed in Table 2. Recently, Gu and Chen [30] pointed out that the inclination parameters slightly varied from brittle to ductile materials. In their extended models of Puck’s theory, the predictions were in good agreement with experimental data particularly for UD composite laminates with high $Y_C/Y_T$ ratios. As the difference between the parabolic and extended models is quite small, the parabolic model is preferred in this work while the inclination parameters are slightly updated according to the results in [30].

3.1. In situ strength effect

The in situ strength effect refers to the fact that the strengths of a single laminar embedded in multi-directional laminates are higher than those of unidirectional laminates obtained by classic material tests. This is because crack initiation and propagation in a single laminar are delayed by adjacent plies with different properties like ply thickness or ply angles. Existing results show that the in situ effect should be carefully taken into account, especially for transverse tension and in-plane shear strengths [31–34].

With novel experimental design and data reduction techniques, the mechanism of in situ strength effect has been well studied and explained with more experimental data and observations revealed [35–37]. A variety of theoretical analysis models have been proposed to quantify the effects, which were subsequently applied in numerical studies to predict the in situ strength [6,9,38–40].

Chang and Lessard [33] introduced an empirical formula for calculating the in situ transverse tension and in-plane shear strengths with several fitting parameters obtained using the reverse method. With
amounts of experimental data on specific material systems and layups, simple multiple values of the corresponding strengths have been recommended to represent in situ strengths [41,42]. Wang and Karthikeyan [34,35] advanced a general theoretical solution for in situ strengths based on fracture mechanics, with full consideration on the constraining effect of adjacent plies. Recently, Dong et al. [39] promoted the solution to more CFRP laminates for further validating its feasibility and reliability. Camanho et al. [9] studied the influence of embedded laminate thickness and extended the original solution. For both accuracy and simplicity, the in situ strengths in this work are calculated using the solution proposed by [39], given by:

\[
\begin{align*}
Y_1^\parallel &= Y_T \left[ 1 + \frac{f_1(\Delta \phi)}{f_1(\Delta \phi)} \right] \\
S_1^\parallel &= S_{12} \left[ 1 + \frac{f_1(\Delta \phi)}{f_2(\Delta \phi)} \right]
\end{align*}
\]  
(16)

where \(Y_1^\parallel\) and \(S_1^\parallel\) are the in situ transverse strength and in-plane shear strength, and \((A, B, C, D)\) are fitting parameters determined by the layups. Wang and Karthikeyan [34] discussed the most applied layups and the likely values of \((A, B, C, D)\). The values (1.7, 3.4 4.0 1.0) were recommended when experimental data were absent. \(N\) is the number of plies in the calculated unidirectional laminar, representing the size effect of the thickness. \(f_1(\Delta \phi)\) and \(f_2(\Delta \phi)\) denote the constraining effects of adjacent plies, which are determined as:

\[
\begin{align*}
f_1(\Delta \phi) &= \min \left[ \frac{\sin^2(\Delta \phi_i)}{1 + \sin^2(\Delta \phi_i)}, \frac{\sin^2(\Delta \phi_i)}{1 + \sin^2(\Delta \phi_i)} \right] \\
f_2(\Delta \phi) &= \min \left[ \frac{\sin^2(\Delta \phi_i)}{1 + \sin^2(\Delta \phi_i)}, \frac{\sin^2(\Delta \phi_i)}{1 + \sin^2(\Delta \phi_i)} \right]
\end{align*}
\]  
(17)

where \(\Delta \phi_i\) is the ply angle difference between the calculated laminate and the very neighboring upper or lower one.

### 3.2. Determination of fracture angle

From Eqs. (3) and (9), the failure is determined by the fracture angle in addition to the stress states. Mathematically, the maximum value of the left part of Eq. (9) corresponds to the potential fracture plane and angle, given by:

\[
f_{\text{IFF}}(\beta_f) = \max \{ f_{\text{IFF}}(\sigma_{12}, \sigma_{13}, \tau_{11}, \tau_{13}, \tau_{23}, \beta) \}, (\beta \in [-90^\circ, 90^\circ])
\]  
(18)

where \(f_{\text{IFF}}\) is the failure index representing the left part of Eq. (9).

Enumeration algorithm was firstly introduced to determine \(\beta_f\) [2,17]. This leads to testing all the angles individually at an interval of \(\Delta \phi\), with a high computational effort. To reduce testing points, Davila et al. [7] and Pinho et al. [6] estimated the potential fracture angle using trial angles and drawing trial failure envelopes. Golden section search algorithm (GSSA) was further introduced and extended to significantly improve the computational efficiency [29,43,44].

In GSSA, one unknown point is generated by two known points, with the distance satisfying the golden section rule as schematically illustrated in Fig. 6. For a stress state, \((\beta_1, f_{\text{IFF}}(\beta_1))\) and \((\beta_2, f_{\text{IFF}}(\beta_2))\) are two preset points, then two new points \((\beta_3, \beta_4)\) can be determined as:

\[
\begin{align*}
\beta_3 &= \frac{\beta_2 + \beta_1}{2}, \quad \beta_4 = \beta_1 + \frac{\sqrt{5} - 1}{2} \beta_2 \\
\beta_3 &= \beta_1 + \frac{\sqrt{5} - 1}{2} \beta_2
\end{align*}
\]  
(19)

In case \(f_{\text{IFF}}(\beta_1) > f_{\text{IFF}}(\beta_2)\), the new search sub-range is located at \([\beta_2, \beta_3]\), otherwise \([\beta_4, \beta_1]\). After several search trials, a parabola can be constructed as the target function to reduce iterations and searching time [29], yielding.
where $R^k(\phi)$ and $R^m(\theta)$ are the transformation matrices defined as:

$$R^k(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & \sin \phi \\ 0 & -\sin \phi & \cos \phi \end{bmatrix}, \quad R^m(\theta) = \begin{bmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{bmatrix}$$

Hence, stresses at $1-2-3$ system can be projected into $m_1 - m_2 - m_3$ and further onto the matrix fracture plane, which gives

$$\sigma_{kk}^{mk} = \sigma_{kk}^{fr} \cos^2 \phi + \sigma_{kk}^{fr} \sin^2 \phi + 2\sigma_{kk}^{fr} \cos \phi \sin \phi$$

$$\tau_{kt}^{mk} = \frac{\tau_{kt}^{fr}}{R^m(\theta)} + \frac{\tau_{kt}^{fr}}{R^m(\phi)}$$

$$\tau_{tf}^{mk} = -\sigma_{kk}^{fr} \cos \phi \sin \phi + \sigma_{kk}^{fr} \cos \phi \sin \phi + \frac{\tau_{kt}^{fr} \cos^2 \phi - \sin^2 \phi}{R^m(\phi) R^m(\theta)}$$

(24)

Then the local matrix failure is determined using inter-fiber failure criteria with applied kinking stresses, as:

$$\sqrt{\left( \frac{1}{R^m(\theta)} \right)^2 + \left( \frac{1}{R^m(\phi)} \right)^2 + \left( \frac{\phi_f}{R^m(\theta) R^m(\phi)} \right)^2} = 1, \quad \sigma_{kk}^{mk} \geq 0$$

$$\sqrt{\left( \frac{1}{R^m(\theta)} \right)^2 + \left( \frac{1}{R^m(\phi)} \right)^2 + \left( \frac{\phi_f}{R^m(\theta) R^m(\phi)} \right)^2} = 1, \quad \sigma_{kk}^{mf} < 0$$

(25)

4.2. Solution for kinking band parameters

From the formulation of fiber kinking model, the determination of angle parameters ($\phi$, $\theta$) is crucial to predict the longitudinal compression failure.

During the crack formation, fiber defects result in shear stiffness decreasing in the kink bands. The out-of-plane shear stress ($\tau_{tf}^{mk}$) on the kinking plane is assumed to be zero. In fact, if $\tau_{tf}^{mk} \neq 0$, fibers would keep deflecting perpendicular to the kinking plane, contradicting the current configuration [6]. From Eq. (23), the kinking plane angle $\phi$ can be derived as:

$$\phi = \frac{1}{2} \arctan \left( \frac{2\tau_{tf}^{mk}}{\sigma_{kk}^{fr} - \sigma_{kk}^{mk}} \right)$$

(26)

Relation between fiber misalignment angle ($\theta$) and shear strain in the local misalignment plane ($\gamma_{12}^{m}$) is shown in Fig. 8, as:

$$\theta = \frac{\tau_{12}^{m}}{\gamma_{12}^{m} + \beta_{b}}$$

(27)

where $\gamma_{12}^{m}/\tau_{12}^{m}$ denotes the consideration on directions of initial misalignment angle, namely, $\pm \beta_{b}$. $\gamma_{12}^{m}$ can be determined by the shear strain-stress constitutive relationship, which can be briefly expressed as:

$$\tau_{12}^{m} = f(\gamma_{12}^{m})$$

(28)

Therefore, at a general 3D stress state, combining the off-axis stress transformation in Eq. (23), gives:

$$f(\gamma_{12}^{m}) = -\sin(\theta) \cos(\theta) (\sigma_{kk}^{fr} - \sigma_{kk}^{mk}) + (\cos^2(\theta) - \sin^2(\theta)) \tau_{tf}^{mk}$$

(29)
For sufficiently small values of $\delta$, Eq. (29) can be approximated as,

$$f(\gamma_{12}^m) \approx -(\sigma_{11}^m - \sigma_{22}^m)\theta + |\tau_{12}^m|$$

Specifically, for linear shear response, shear strain in the local misalignment plane $\gamma_{12}^m$ can be simplified as,

$$\gamma_{12}^m = \frac{\partial_0 G_{12} + |\tau_{12}^m|}{G_{12} + \sigma_{11}^m - \sigma_{22}^m} - \delta_0$$

On the other hand, with nonlinear shear response taken into account, the Newton-Raphson iteration method is applied to solve Eq. (30) with the additional mathematical condition, as:

$$\frac{\partial f(\gamma_{12}^m)}{\partial \gamma_{12}^m} = -(\sigma_{11}^m - \sigma_{22}^m)\cos 2\theta - 2 |\tau_{12}^m| \sin 2\theta$$

5. Degradation scheme of material properties

At a stress state, in case that one of the failure criteria is satisfied as stated in previous sections, the corresponding material property is supposed to be degraded in order to characterize the decrease of structural load bearing capacity. The left hand terms of Eqs. (1), (9), (25) are denoted as $f_i$, $(i = t, IFFT, IFCC, kink)$. The degradation differs for different failure modes and is listed below.

(1) Fiber tension failure. The failure caused by fiber breakage is generally instantaneous and catastrophic, leading to massive loss in load bearing capacity. Therefore, when fiber tension failure is triggered, all the in-plane mechanical properties are simply degraded to zero, i.e.,

$$\{E_{11}, E_{22}, G_{12}, v_{12}\}_t \rightarrow [0, 0, 0, 0].$$

(2) Inter-fiber failures. In case that the normal traction on the fracture plane is non-negative ($\sigma_n \geq 0$), inter-fiber tensile failure (IFFT) occurs. The fracture plane is usually perpendicular to the loading direction, causing an crack to remain open. It leads to a certain degradation in the transverse elastic and inter-fiber shear modulus as well as the Poisson’s ratio, expressed as:

$$\{E_{22}, G_{12}, v_{12}\}_{IFFT} \rightarrow \{\eta_{IFFT}E_{22}, \eta_{IFFT}G_{12}, \eta_{IFFT}v_{12}\}.$$  

(3) Fiber kinking failure. In case that kink bands are formed, the corresponding laminar can hardly carry any load since amounts of fiber misalignments and localized matrix failures exist in the kink bands. Therefore, all the in-plane mechanical properties are simply degraded to zero, namely,

$$\{E_{11}, E_{22}, G_{12}, v_{12}\}_{IFC} \rightarrow [0, 0, 0, 0].$$

6. Verification and discussion

Fig. 9 presents the predicted $\sigma_{22} - \gamma_{12}$ failure envelopes of glass- and carbon-fiber reinforced unidirectional (UD) laminates, namely, E-Glass/LY556 [27], AS4/55A [49], T800/3900 [50] and AS4/3501 [51]. The material properties are listed in Table 3. It is observed that the present predictions correlates well with existing experimental data.

Both the experimental data and theoretical predictions show that the maximum shear stress $\tau_{12}$ increases with increasing $|\sigma_{22}|$ at low level of transverse compression. This indicates that transverse compression can inhibit shearing damage to a certain extent. In this case, in-plane shear failure occurs mainly in UD laminates with the fracture angle located between $0^\circ$ and $40^\circ$ [7]. For higher transverse compression ($|\sigma_{22}| > 0.7X_C$), shearing effect is less on the failure, and the UD
laminates fail mostly in compressive behavior. The proposed model captures this phenomenon, further validating its feasibility.

Hinton et al. [27] reported experimental data of T300/BLS914C UD laminates under combined loading status $\sigma_{11} - \tau_{12}$. The mechanical properties T300/BLS914C are listed in Table 1.

Fig. 10 shows a good agreement between the predicted failure envelope and experimental data points. It should be mentioned that in-plane shear nonlinearity is taken into account by using the cubic spline interpolation function [52]. In the case of high level tension ($\sigma_{11} \geq 1000$ MPa), shearing behavior has less influence on fracture mode. The quadratic Hashin criterion provides well-fitted predictions, with test data points distributed around both sides of the failure envelope. Similar to transverse compression, for low levels of $|\sigma_{11}|$ (e.g. $-400$ MPa < $\sigma_{11}$ < 0 MPa), fiber compression slightly enhances the shear fracture resistance. Both the predictions and test data capture this phenomenon as shown in Fig. 10. This finding may be further applied to prestressed composite structures in engineering applications. Regarding the pure shearing state ($\sigma_{11} = 0$), it is noted that a high dispersion of data points is observed in Fig. 10. It has been suggested that experimental errors caused the higher values of some data in comparison to the shear strength [3,12]. For this consideration, no special emphasis is given in this study. Accordingly, it can be concluded that the proposed model can give a satisfactory prediction of the combined $\sigma_{11} - \tau_{12}$ failure.

The predicted bi-axial $\sigma_{11} - \sigma_{22}$ failure envelope of E-Glass/MY750 UD laminates is illustrated in Fig. 11. Experimental data were employed in WWFE [3,27]. Relevant mechanical properties of E-Glass/MY750 can be found in Table 4.

As can be noted in Fig. 11, the four interaction points of the failure envelope and $\sigma_{11} - \sigma_{22}$ axes exactly correspond to the four basic strengths $X_X, X_C, Y_Y, Y_C$ obtained from tests. In the fourth quadrant, however, the test data are more conservative than the predictions. This may be attributed to the fact of that experimental data were obtained by $\pm 5^\circ$ rather than $0^\circ$ UD laminates. In the second and third quadrants, the failure envelope shows that longitudinal compression slightly reduces the transverse tension and compression resistance, though few test data were obtained from the combined loading. On the other hand, the predictions fit well with experimental results in the first quadrant in bi-axial tension. To further examine the proposed model, more tests should be performed in the future.

Soden et al. [16] presented several experimental tests of multi-directional composite laminates under uniaxial and biaxial loads in WWFE. Some of experimental results are employed in this work to further validate the proposed model as shown in Figs. 12 and 13. Fig. 12 displays the strain-stress responses of orthogonal E-Glass/MY750 laminates ($[0/90]$) under uniaxial tension. Predictions with and without considering in situ strengths are both demonstrated. In general, a good agreement of the strain-stress curves is observed between the prediction and test data, regardless of the consideration of in situ strengths. And both predictions capture initial failure in the 90° layer, indicating that in situ effects have little influence on the loading path. However, the initial failure stress predicted by the model without in situ strengths is only 80 MPa, which is notably lower than the test result (159 MPa). With the in situ strength accounted for, the predicted initial failure stress is 150 MPa, correlating well with the test result as shown in Fig. 12 and Table 5.

In the case of biaxial tension ($\sigma_{11} - \sigma_{22}$), the strain-stress responses for quasi-isotropic AS4/3501 laminates ($[90/\pm45/0]$) are illustrated in Fig. 13. Similarly, compared to that using the basic strengths, the initial failure stress predicted using the in situ strengths is closer to the test result, with a deviation of 8.8% as is seen in Table 5. This proves the importance of considering in situ strengths in failure analysis. The consistency of the predicted strain-stress responses with experimental data points after initial failure further validates the feasibility of the

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**Table 3**

Material properties of investigated composites in terms of $\sigma_{11} - \tau_{12}$ failure envelopes [49,27,50,51].

<table>
<thead>
<tr>
<th>E-Glass/LY556</th>
<th>AS4/55A</th>
<th>T800/3900</th>
<th>AS4/3501</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{11}$/GPa</td>
<td>54</td>
<td>120</td>
<td>175</td>
</tr>
<tr>
<td>$E_{22}$/GPa</td>
<td>18</td>
<td>12</td>
<td>10</td>
</tr>
<tr>
<td>$\gamma_{12}$</td>
<td>0.28</td>
<td>0.28</td>
<td>0.30</td>
</tr>
<tr>
<td>$G_{12}$/GPa</td>
<td>5.8</td>
<td>6.5</td>
<td>5.2</td>
</tr>
<tr>
<td>$X_X$/MPa</td>
<td>1140</td>
<td>1950</td>
<td>2000</td>
</tr>
<tr>
<td>$X_C$/MPa</td>
<td>570</td>
<td>1480</td>
<td>1500</td>
</tr>
<tr>
<td>$Y_Y$/MPa</td>
<td>35</td>
<td>50</td>
<td>60</td>
</tr>
<tr>
<td>$Y_C$/MPa</td>
<td>114</td>
<td>200</td>
<td>201</td>
</tr>
<tr>
<td>$\gamma_{12}$/MPa</td>
<td>72</td>
<td>79</td>
<td>100</td>
</tr>
<tr>
<td>$\beta_{12}$</td>
<td>53</td>
<td>53</td>
<td>53</td>
</tr>
</tbody>
</table>

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**Table 4**

Mechanical properties of E-Glass/MY750 unidirectional laminates [16].

<table>
<thead>
<tr>
<th>$E_{11}$/GPa</th>
<th>$E_{22}$/GPa</th>
<th>$G_{12}$/GPa</th>
<th>$\gamma_{12}$</th>
<th>$\beta_{12}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>46 GPa</td>
<td>16 GPa</td>
<td>5.83 GPa</td>
<td>0.28</td>
<td>58</td>
</tr>
<tr>
<td>$X_X$</td>
<td>$X_C$</td>
<td>$Y_Y$</td>
<td>$Y_C$</td>
<td>$S_{12}$</td>
</tr>
<tr>
<td>1280 MPa</td>
<td>800 MPa</td>
<td>40 Mpa</td>
<td>145 MPa</td>
<td>73 MPa</td>
</tr>
</tbody>
</table>
proposed material degradation scheme.

Fig. 14 illustrates both the initial and final failure envelopes of a angle-ply laminate ([±55]s), with experimental data taken from [16]. A good agreement is observed between the predicted final failure envelope and experimental data, indicating once again the reliability of the proposed failure model. It is noted that the angle-ply laminate is easier to fail under biaxial tension and compression (σx/σy < 0), compared to its uniaxial strengths as shown in the second and fourth quadrants of Fig. 14. On the other hand, both the predictions and experimental data in the first and third quadrants show that biaxial tension and biaxial compression enhance both the uniaxial tension and compression fracture resistance of the angle-ply laminate.

Particularly for biaxial compression in the third quadrant, the predicted failure envelope provides conservative results relative to some of the test data (e.g. σx < −300 MPa). This might be the result of that the fiber volume fraction of biaxial compression specimens (68%) was not consistent with that of biaxial tension ones (60%) in tests [3]. And the material properties used in the present failure model are based on those of unidirectional laminates with the fiber volume fraction fixed at 60% in accordance with the literature [16].

7. Conclusions

In this work, a physically-based failure analysis framework is proposed to predict the intralaminar failure and strengths for composite laminates, including fiber tension, inter-fiber failure (IFF) and fiber kinking failure. In 3D stress states, superposition and coupling effects are carefully considered for multiaxial loads. The in situ strengths are introduced into the failure model using fracture mechanics-based approximation formula. The size effect of embedded laminar thickness and the constraining effects of adjacent plies are taken into account simultaneously. For CFRP and GFRP laminates with thin fibers and high fiber volume fractions (40%-60%), a longitudinal fiber kinking model is established with initial manufacturing defects considered. To characterize the decrease of load bearing capacity and final failure, a simplified degradation scheme of material properties is proposed, differing from different failure modes.

Predicted failure envelopes of various laminates under multiaxial loads are illustrated, like unidirectional (UD), orthogonal, quasi-isotropic and angle-ply laminates. Good agreement is observed comparing the predicted initial and final failure envelopes with experimental data. From both experimental data and predictions, it is further found that slight compression could enhance the in-plane shear fracture resistance of UD laminates, whereas biaxial tension and biaxial compression enhance the uniaxial tension and compression strengths of angle-ply laminates.

The influence of in situ strengths on initial failure stress and strain-stress responses is discussed in detail. Results show that predictions obtained using basic strengths considerably underestimated the initial

Table 5

<table>
<thead>
<tr>
<th>In situ effects on initial failure stresses.</th>
<th>Experiments</th>
<th>without in situ effects</th>
<th>with in situ effects</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0/90], E-Glass/MY750</td>
<td>159 MPa</td>
<td>80 MPa (−49.6%)</td>
<td>150 MPa (−5.6%)</td>
</tr>
<tr>
<td>[90/±45/0], AS4/3501</td>
<td>439 MPa</td>
<td>260 MPa (−40.7%)</td>
<td>400 MPa (−8.8%)</td>
</tr>
</tbody>
</table>
failure stress while the model with in situ effects considered give a good agreement with test data. Strain-stress responses are well captured and consistent with experimental data. This further validates the feasibility of the proposed material degradation scheme. On the other hand, like most failure theories for fiber-reinforced composite laminates, this proposed framework does not consider the possible interfacial failure mode although the contribution of all stresses to interlaminar are addressed. Interlaminar delamination and fiber-matrix debonding may take place following transverse cracking or fibre breakage. However, we have not taken the interface effect into consideration in current model and thus we cannot exactly show how it affects the results in this paper. The work of improving the model would be done in the future.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgment

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